Advanced Functionally Graded Plate-Type Structures Impacted By Blast Loading

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Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
05 AUG 2010	**N/A**	**-**

4. TITLE AND SUBTITLE	5a. CONTRACT NUMBER
Advanced Functionally Graded Plate-Type Structures Impacted By Blast Loading	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)	5d. PROJECT NUMBER
Terry Hause, Ph.D.	5e. TASK NUMBER
	5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
US Army RDECOM-TARDEC 6501 E 11 Mile Rd Warren, MI 48397-5000, USA	**21050**

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
US Army RDECOM-TARDEC 6501 E 11 Mile Rd Warren, MI 48397-5000, USA	**TACOM/TARDEC**
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)
	21050

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
Presented at NDIAs Ground Vehicle Systems Engineering and Technology Symposium (GVSETS), 17 22 August 2009,Troy, Michigan, USA, The original document contains color images.

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**SAR**	**39**	

The author would like to express thanks to the U.S. Army RDECOM TARDEC for their support and funding under the Independent Laboratory In-House Research Program (ILIR).

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1. Motivation
2. Basic Assumptions and Preliminaries
3. Types of FGM Models
4. Theoretical Developments
5. Solution Methodology
6. Blast Loading
7. Results
8. Conclusions

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- A longer service life of the structure operating in extreme environments consisting of severe thermal and mechanical loading conditions.

- Combine the properties of two dissimilar Materials

- The load carrying capacity of plate-type structure is enhanced.

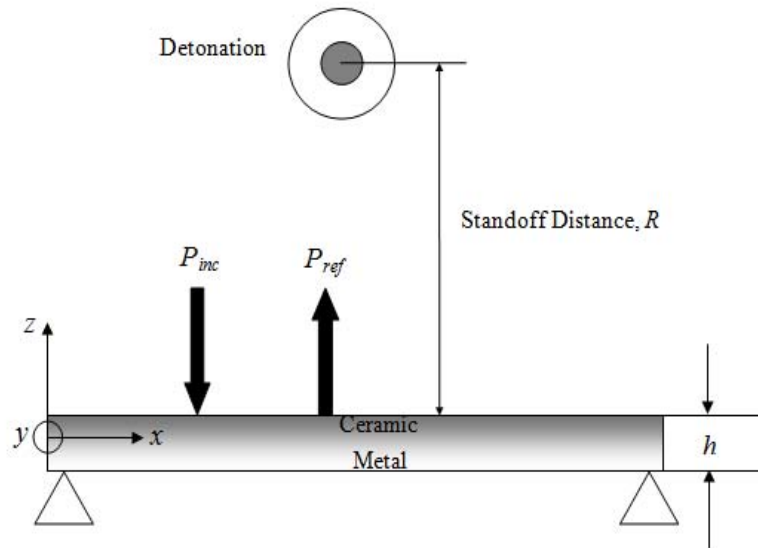- An increased operational life of the structure.

Figure 1. A simply Supported functionally graded (FG) plate shown in 2-D, under an explosive blast

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Basic Assumptions

- The plate mid surface is referred to a Cartesian Orthogonal System of Coordinates (*x, y, z*).

- *Z* is the thickness coordinate measured positive in the upwards direction from the mid-surface of the plate with *h* being the uniform plate thickness.

- The plate is assumed to be thin. As a result, the Kirchoff Assumptions, of zero transverse shear stresses, will be assumed leading to the classical plate theory.

- The functionally graded plate is composed of ceramic and metal phases whose properties vary smoothly and continuously across the wall thickness from one surface to another.

Based on a rule of mixtures, the following form of the variation of mechanical properties (Young's Modulus, Density, and Poisson's Ratio) are postulated as

$$P(z) = (P_{\text{ceramic}} - P_{\text{metal}})V_c(z) + P_{\text{metal}}$$

Denotes a Generic Property

Volume Fraction

Specific Values of the Respective Properties

$$\text{Note}: \quad \text{For } V_c(z) = 0, P(z) = P_m \text{ and For } V_c(z) = 1, P(z) = P_c$$
$$\Rightarrow V_c(z) \in [0,1]$$

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Type 1: Symmetric Distribution of the Constituent Phases

The phases vary symmetrically through the wall thickness in the sense of having full ceramic at the outer surfaces of the plate and tending toward full metal at the mid-surface

$$V(z,N) = \left(\frac{z}{h/2}\right)^N \frac{1+\text{sgn}z}{2} + \left(\frac{z}{-h/2}\right)^N \frac{1-\text{sgn}z}{2}$$

Where,

$$\text{sgn}(z) = \begin{cases} 0, & \text{if } z = 0 \\ -1, & \text{if } z < 0 \\ +1, & \text{if } z > 0 \end{cases} \implies \text{Signum Function}$$

$$N \equiv \text{Volume Fraction Index } (0 \leq N \leq \infty)$$

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MSTV
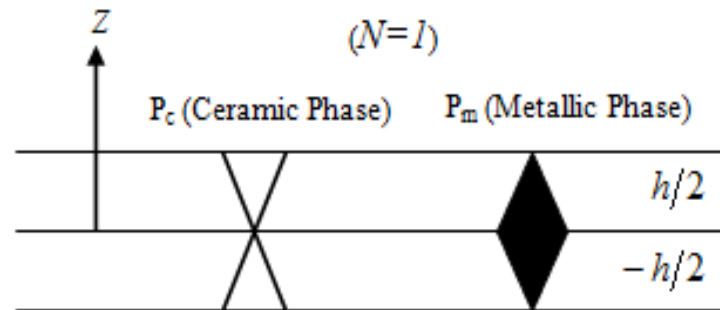MODELING AND SIMULATION, TESTING AND VALIDATION

Figure 2. Distribution of the constituent materials through
The plate thickness for the symmetric case.

at $z = \pm h/2$, for any N, $V = 1$, and as a result $P(\pm h/2) \Rightarrow P_c$, and for $z = 0$, $V = 0$ and $P(0) \Rightarrow P_m$.

Type 2: Asymmetric Distribution of the Constituent Phases

The phases vary non-symmetrically through the wall thickness and in this case there is Full ceramic at the outer surface of the plate wall and full metal at its inner surface.
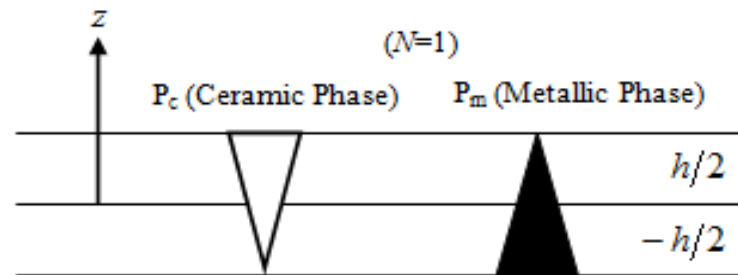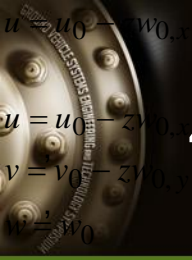
$$V(z, N) = [(h - 2z)/2h]^N$$

Figure 3. Distribution of the constituent materials through
The plate thickness for the asymmetric case.

at $z = +h/2, P(h/2) \Rightarrow P_m$, and for $z = h/2, P(h/2) \Rightarrow P_c$. At the Midsurface,

$z = 0$, and for $k = 1, P(0) = \dfrac{1}{2}(P_c + P_m)$

3-D Displacement Field

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial x}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0(x,y,t)}{\partial y}$$

$$w(x,y,z,t) = w_0(x,y,t)$$

$u_0, v_0, w_0 \Rightarrow$ 2-D displacement quantities of the mid-surface

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Non-linear Strain-Displacement Relationships

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

$$\varepsilon_{xz} = \varepsilon_{yz} = \varepsilon_{zz} = 0$$

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Substitution of the displacement field into the strain-displacement relationships Gives the 3-D strain measures across the plate thickness as

$$
\left\{ \begin{array}{c} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{array} \right\} = \left\{ \begin{array}{c} \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\[2mm] \dfrac{\partial v_0}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\[2mm] \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} \end{array} \right\} + z\left\{ \begin{array}{c} -\dfrac{\partial^2 w_0}{\partial x^2} \\[2mm] -\dfrac{\partial^2 w_0}{\partial y^2} \\[2mm] -\dfrac{\partial^2 w_0}{\partial x \partial y} \end{array} \right\}
$$

$\{\varepsilon\}$

3D Strain Measures

$\left\{\varepsilon^{(0)}\right\}$

2D in-plane strain measures

$\left\{\varepsilon^{(1)}\right\}$

2D bending Strains

Plane Stress Constitutive Equations

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\sigma_{xz} = \sigma_{yz} = \sigma_{zz} = 0$$

Where,

$$Q_{11} = Q_{22} = \frac{E(z)}{1-v^2}, \ Q_{12} = \frac{vE(z)}{1-v^2}, \ Q_{66} = \frac{E(z)}{2(1+v)}, \ Q_{16}, Q_{26} = 0$$

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Governing Equations

$$\int_{t_0}^{t_1} (\delta U + \delta V - \delta K)dt = 0 \quad \Rightarrow \text{Hamilton' s Principle}$$

t_0, t_1 are two arbitrary instants in time

U, denotes the strain energy

V, denotes the work done by surface tractions, edge loads, and
     body forces

K, denotes the kinetic energy of the 3 - D body of the Structure

δ, denotes the variational operator

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Strain Energy

$$\delta U = \int_{\Omega_0} \int_{-h/2}^{h/2} (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{xy}\delta\gamma_{xy})dzd\Omega_0$$

External Work

$$\delta V = -\int_{\Omega_0} P_t(x,y)\delta w\left(x,y,\frac{h}{2}\right)d\Omega_0 - \int_x \int_{-h/2}^{h/2}\left(\sigma_{yy}^*\delta v + \sigma_{yx}^*\delta u\right)dzdx -$$

$$\int_y \int_{-h/2}^{h/2}\left(\sigma_{xx}^*\delta u + \sigma_{xy}^*\delta v\right)dzdy$$

Kinetic Energy

$$\delta K = \int_{\Omega_0} \int_{-h/2}^{h/2} \rho(z)\dot{W}\delta\dot{W}dzd\Omega_0$$

Equations of motion in terms of Stress resultants

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_{yy}}{\partial y^2} + \frac{\partial}{\partial x}\left(N_{xx}\frac{\partial w_0}{\partial x} + N_{xy}\frac{\partial w_0}{\partial y} \right) +$$

$$\frac{\partial}{\partial y}\left(N_{yy}\frac{\partial w_0}{\partial y} + N_{xy}\frac{\partial w_0}{\partial x} \right) + P_t - C\dot{w}_0 = I_0\ddot{w}_0$$

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Where,

$$\left(N_{ij}, M_{ij}\right) = \int_{-h/2}^{h/2} \sigma_{ij}(1, z)\,dz \qquad (i, j = x, y, xy) \Rightarrow \text{Stress Resultants} \quad \text{and}$$

$$\text{Stress Couples}$$

Simply Supported Boundary Conditions

$$w_0 = M_{xx} = N_{xy} = 0, N_{xx} = N_{xx}^* \text{ on } x = 0, L_1$$

$$w_0 = M_{yy} = N_{yx} = 0, N_{yy} = N_{yy}^* \text{ on } y = 0, L_2$$

For Compressive Edge Loading, $N_{xx}^* = -N_{xx}^0$ and $N_{yy}^* = -N_{yy}^0$

The first two equations of motion are satisfied by assuming the following

$$N_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \, N_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \, N_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

It can be shown, the details of which are not shown here, that the third equation of motion can be expressed in terms of displacements and a stress potential as

$$D\nabla^4 w_0 - \left(\frac{\partial^2 \phi}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2} - 2 \frac{\partial^2 \phi}{\partial x \partial y} \frac{\partial^2 w_0}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y^2} \frac{\partial^2 w_0}{\partial x^2} \right) + I_0 \ddot{w}_0 + C\dot{w}_0 = P_t$$

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Where,

$$D = \frac{E_1 E_3 - E_2^2}{E_1(1 - \nu^2)}$$

One more equation in terms of the transversal deflection and the stress potential is needed. This will come from the strain displace-ment relationships, by eliminating the in-plane displacements, which is known as the compatibility equation which can be shown to be given by

$$\nabla^4 \phi = E_1 \left[\left(\frac{\partial^2 w_0}{\partial x \partial y} \right) - \frac{\partial^2 w_0}{\partial x^2} \frac{\partial^2 w_0}{\partial y^2} \right]$$

Where, $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ $\Rightarrow$ Laplacian Operator

Assume a solution form for ϕ, w_0

$$w_0(x, y, t) = w_{mn}(t)\sin\lambda_m x \sin\mu_n y$$

$$\phi = A_{mn}(t)\cos 2\lambda_m x + B_{mn}(t)\cos 2\mu_n y + C_{mn}(t)\cos 2\lambda_m x \cos 2\mu_n y$$

$$+ D_{mn}(t)\sin 2\lambda_m x \sin 2\mu_n y + \frac{1}{2}N_{xx}^* y^2 + \frac{1}{2}N_{yy}^* x^2$$

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Where,

$$\lambda_m = m\pi/L_1, \; \mu_n = n\pi/L_2, \quad m, n = 1, 2, 3, \dots, etc.$$

Utilizing the compatibility equation and comparing coefficients gives

$$A_{mn}(t) = \frac{E_1 w_{mn}^2(t)\mu_n^2}{32\lambda_m^2}, \; B_{mn}(t) = \frac{E_1 w_{mn}^2(t)\lambda_m^2}{32\mu_n^2}, \; C_{mn}(t) = D_{mn}(t) = 0$$

Transversal Pressure

Let,

$$P_t = P_{mn}(t)\sin\lambda_m x \sin\mu_n y$$

By integrating over the plate area gives

$$P_{mn}(t) = \frac{4}{L_1 L_2}\int_0^{L_2}\int_0^{L_1} P_t(t)\sin\lambda_m x \sin\mu_n y\, dx\, dy$$

or

$$P_{mn}(t) = \frac{16P_t(t)}{\pi^2}, \quad \text{Where,} \quad m,n = 1$$

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Finally, applying the Galerkin method to the third equation of motion
Gives the nonlinear equation differential equation governing the
Structural response of FG plates under external excitation.

$$\ddot{w}_{mn}(t) + 2\Delta_{mn}\omega_{mn}\dot{w}_{mn}(t) + \omega_{mn}^{2}w_{mn}(t) + \Omega_{mn}w_{mn}^{3}(t) = \tilde{P}_{mn}(t)$$

$w_{mn}(t) \Rightarrow$ Amplitude of Deflection

$\omega_{mn} = \sqrt{K_{mn}/I_0} \Rightarrow$ The natural frequency of the plate

$\Delta_{mn} = C/2I_0\omega_{mn} \Rightarrow$ Nondimensi onal damping factor

$\Omega_{mn} = E_1(\lambda_m^4 + \mu_n^4)/16$

$\tilde{P}_{mn}(t) = 16P_t(t)/I_0\pi^2$

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$$K_{mn} = \frac{D\pi^4}{L_1^4}(m^4 + 2m^2n^2\psi^2 + n^4\psi^4) + \frac{N_{xx}^*\pi^2}{L_1^2}(m^2 + n^2\Phi\psi^2)$$

Where,

$$\psi = L_1/L_2 \Rightarrow \text{Aspect Ratio}$$

$$\Phi = N_{yy}^*/N_{xx}^* \Rightarrow \text{Compressive/Tensile Edge Load Ratio}$$

The governing nonlinear differential equation is solved using the 4th-Order Runge-Kutta method assuming zero initial conditions.

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For a free in-air spherical air burst, the pressure profile over time is given in figure 4 as
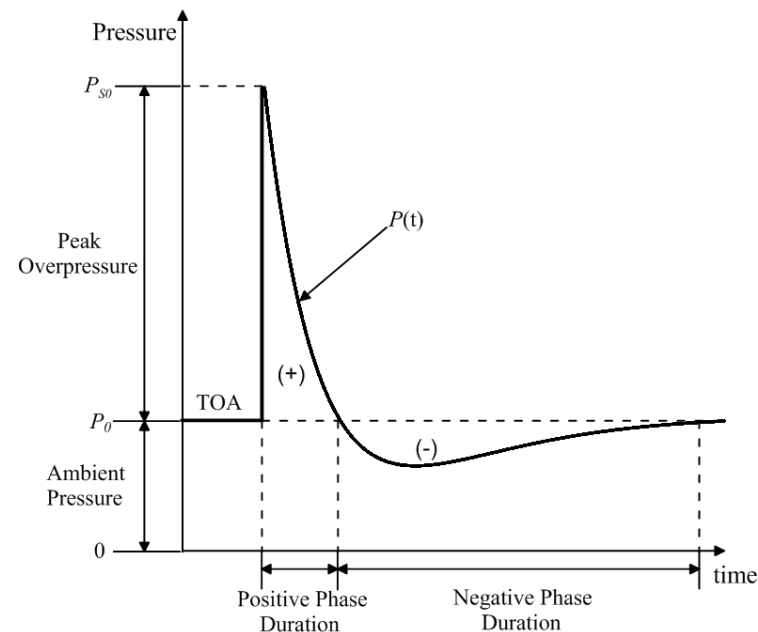
Fig 4. Incident Profile of a blast wave

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The wave form shown in figure 4 is given by an expression known as The Friedlander equation and is give as

$$P_t(t) = (P_{so} - P_o)\left(1 - \frac{t-t_a}{t_p}\right)e^{-\alpha\left(\frac{t-t_a}{t_p}\right)}$$

Where,

$$P_{so} = \frac{1772}{Z^3} - \frac{114}{Z^2} + \frac{108}{Z} \Rightarrow \text{Peak Overpressure over ambient}$$

$$Z = R/W^{1/3} \Rightarrow \text{scaled distance} \begin{cases} R \text{ is the Standoff Distance} \\ W \text{ is the equivalent weight of charge} \\ \text{of TNT in terms of kilograms} \end{cases}$$

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P_o is the ambient pressure

t_a is the time of arrival

t_p is the positive phase duration of the blast wave

t is the time

For conditions of STP at sea level, the time of arrival and the positive phase duration can be determined from

Arrival time or positive phase duration

$$\frac{t}{t_1} = \frac{R}{R_1} = \left(\frac{W}{W_1}\right)^{\frac{1}{3}} \Rightarrow \text{Cube root scaling}$$

Arrival time or positive phase duration for a reference explosion of charge weight, W_1

It should be noted that the standoff distances are themselves scaled According to the cube root law

Validation of the Present Approach

Comparisons are made (Akay, H.U., 1980) who considered explosive blast of Metallic plates under a step loading excitation given by

$$P_t(t) = P[H(t) - H(t - t_0)]$$

$H(t)$ is referred to as the Heaviside Step Function defined as $H(t) = 1$ for $t \geq 0$ and $H(t) = 0$ for $t < 0$

The geometrical and material properties used were

$$L_1 = 2.438\,m, h = 0.00635\,m, \psi = 1$$

$$E_m = 70.3\,GPa, \rho_m = 2547\,kg/m^3, v_{ave} = 0.25$$

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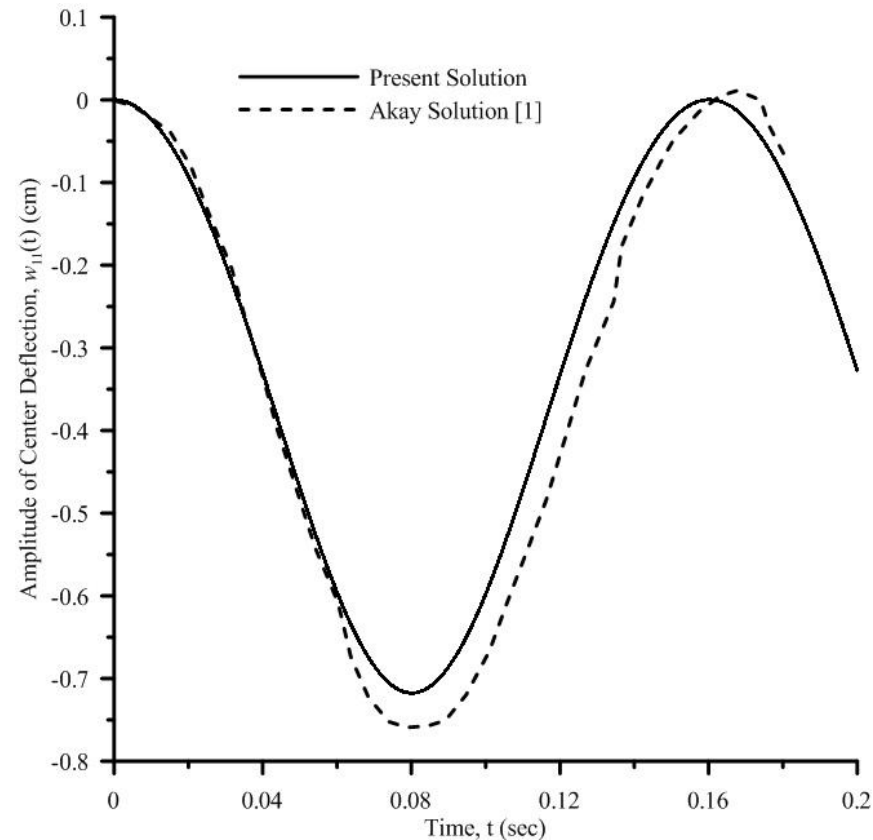
Fig 5. Comparisons of solutions of the time history of the Central deflection Under a Step Load $P = 48.82 Pa$, $t_0 = 0.2\,\text{sec}$

Present Results

A ceramic-metal functionally graded plate with the following properties were used

Table 1. Material Properties

	Metal (Ti-6Al-4V)	Ceramic (Aluminum Oxide)
E, Modulus (GPa)	105.7	320.24
υ, Poisson's Ratio (Unitless)	0.2981	0.26
ρ, Density (Kg/m^3)	4429	3750

($L_1 = 1$ m, $\psi = 1$, $h = 0.0254$ m)

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7. Results-Continued

Table 2. Air blast parameters versus distance for a one Kilogram TNT spherical air burst (Kingery and Bulmash, 1984).

Standoff Distance, (m)	Arrival Time, (msec)	Positive Phase Duration, (msec)
1.0	0.532	1.79
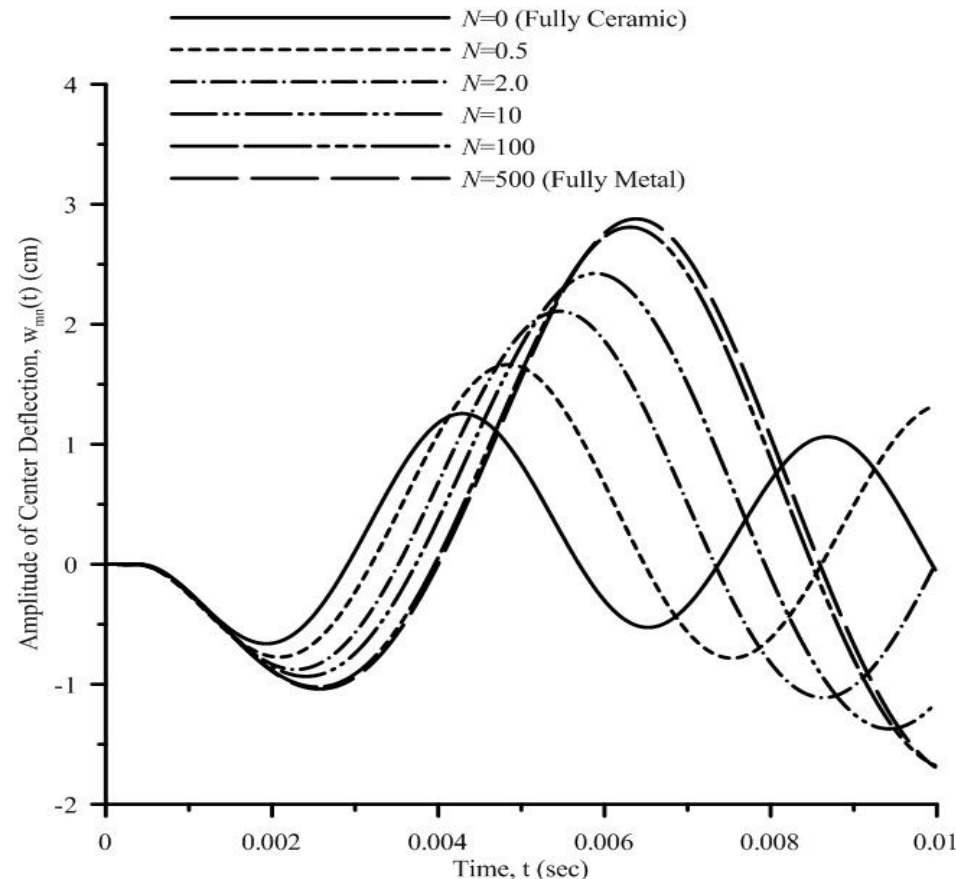
Fig 6. Implications of the volume fraction index on the deflection-time history of an asymmetric FG Plate without damping.
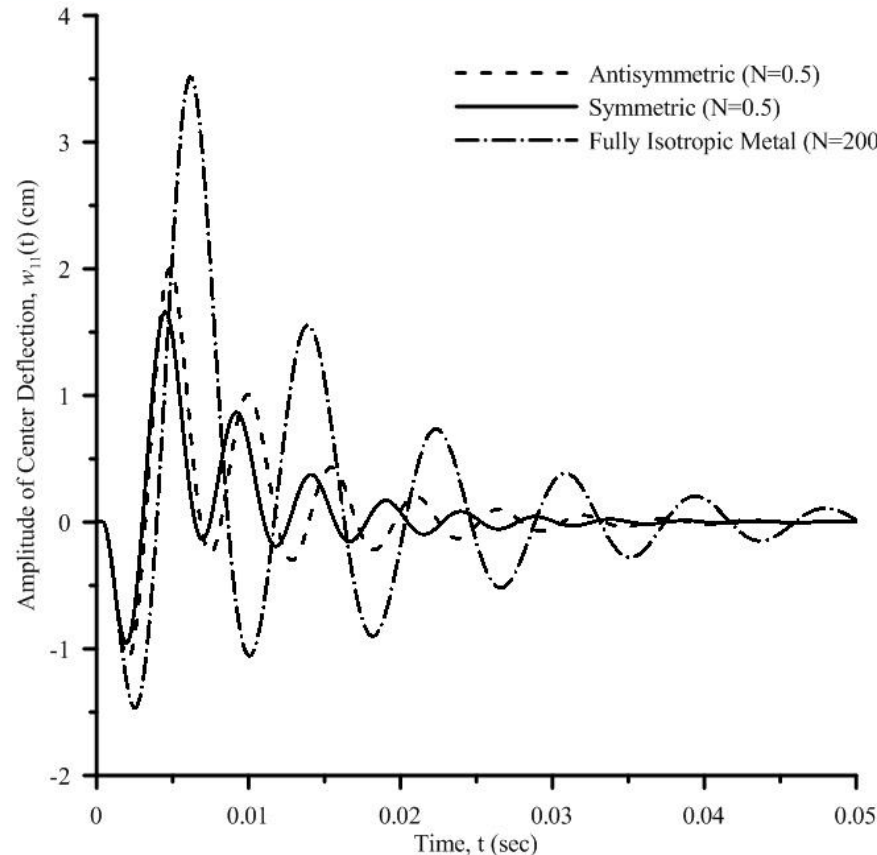
Fig 7. Implications of the symmetry on the deflection-time response of FG plate. $\Delta_{11} = 0.1$
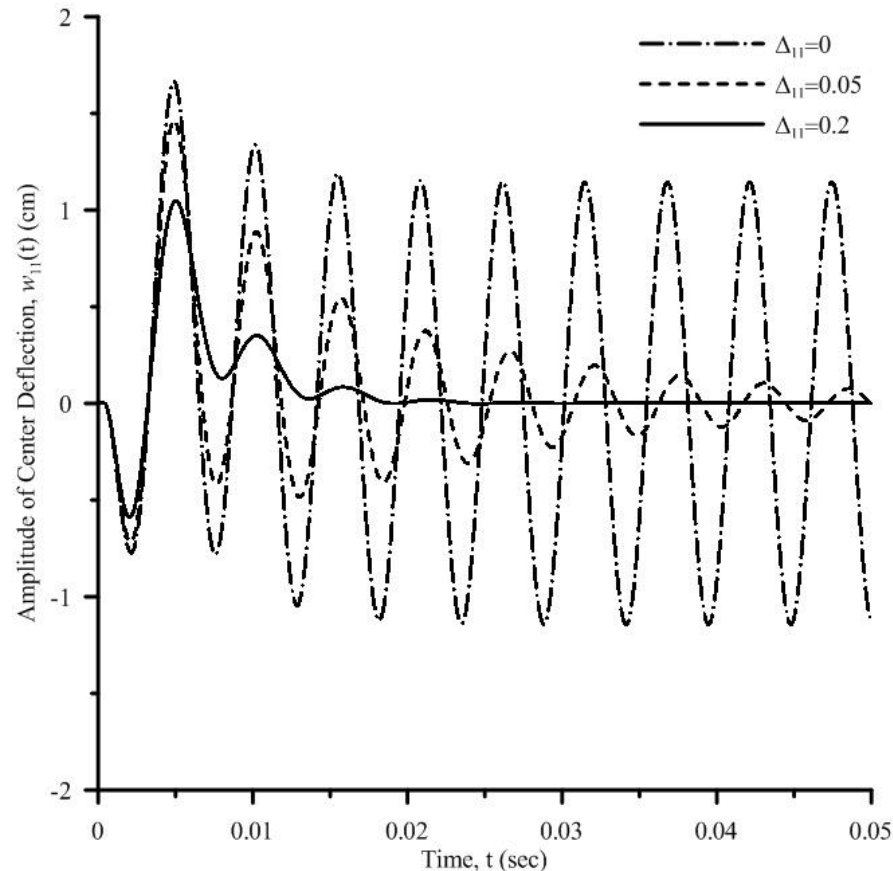
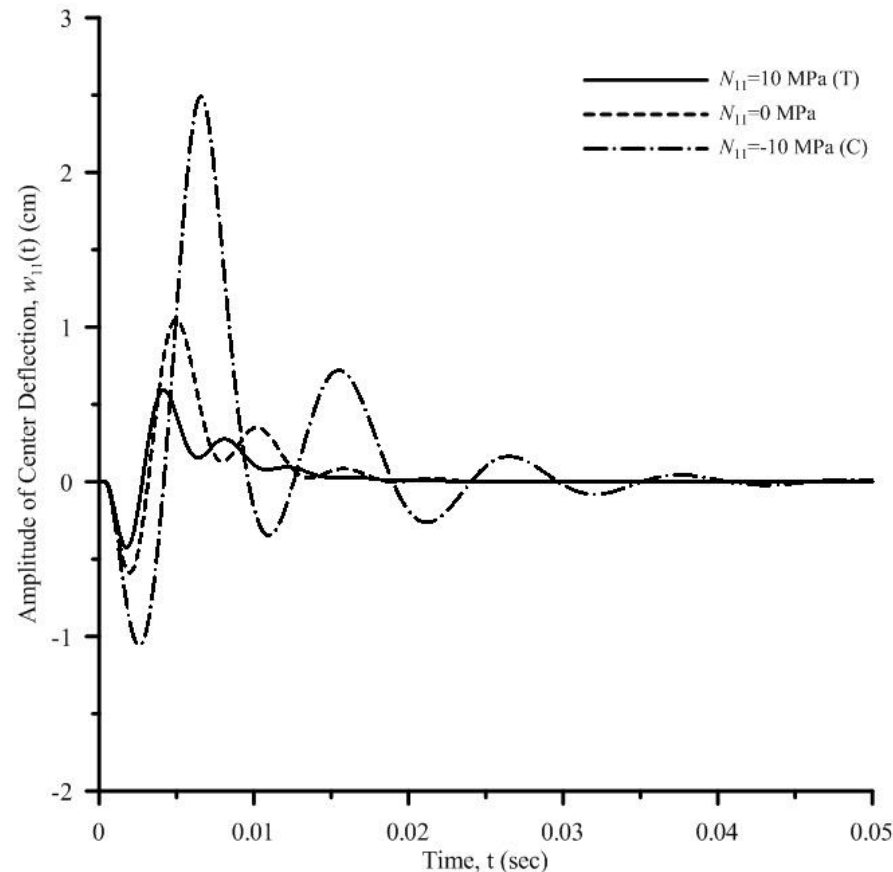Fig 8. Implications of various amounts of damping on the deflection-time response of an asymmetric FG Plate. (N=0.5)

Fig. 9. The implications of the compressive/tensile edge loading
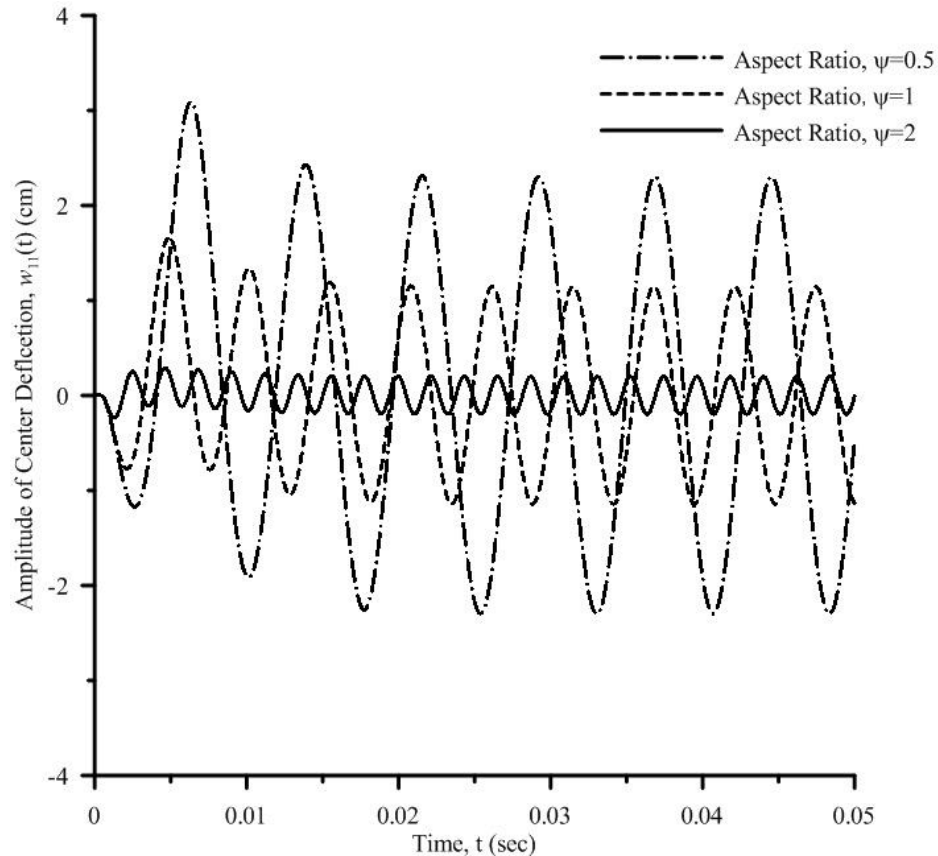on the deflection-time response of an asymmetric FG plate.
$(\Delta_{11} = 0.2, N = 0.5)$

Fig. 10. The effects of various aspect ratios of an asymmetric FG plate on the deflection-time response. (N=0.5)

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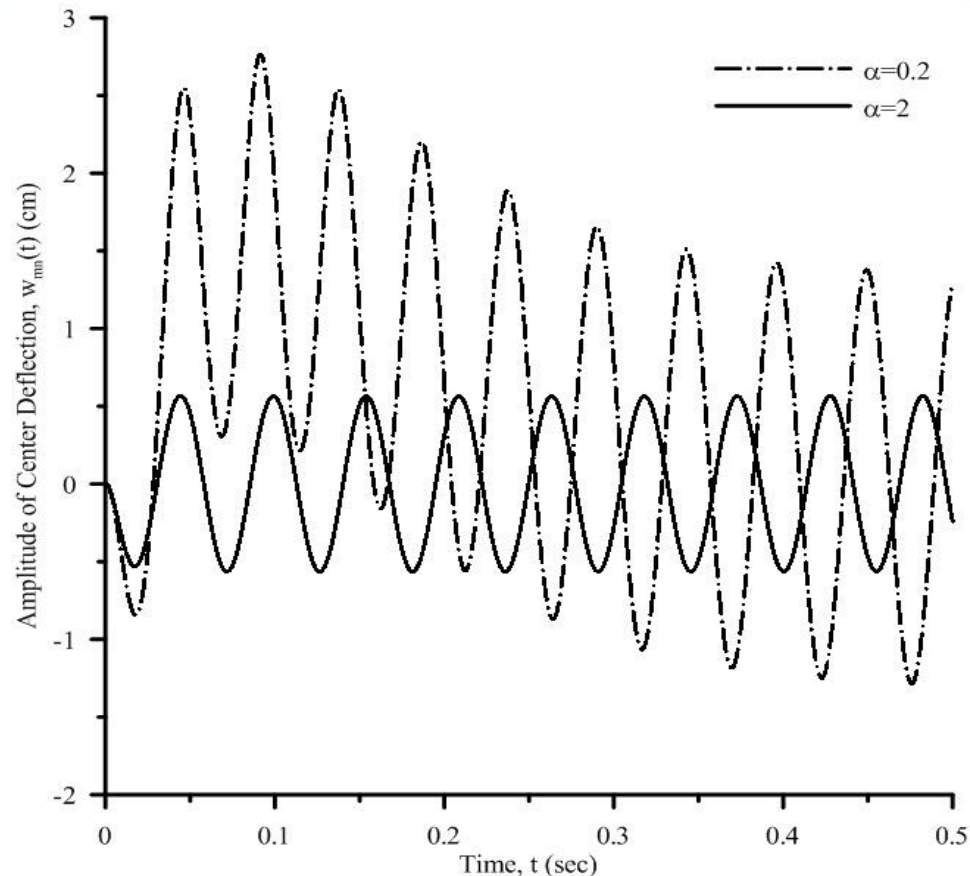

Fig. 11. The implications of the value of the decay parameter on the time-deflection response of a asymmetric FG Plate without damping. (N=0.5)

- A Structural Model of the dynamic response of functionally graded plates exposed to a free in-air blast with simply supported boundary conditions has been presented.

- As the volume fraction of ceramic increases, the deflections become smaller with higher frequencies in contrast to metal phase.

- The symmetric construction due to being stiffer has smaller deflections and higher frequencies as compared to the asymmetric construction

- For a fixed compressive edge load, as the metal concentration increases, the deflections increase with decreasing frequencies.